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Collision of two interfaces of different roughnesses: a numerical study

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Abstract

In the classical Eden model (EM) the growing sites have the same probability of becoming occupied and the growing process in (1 + 1) dimensions leads to the formation of a self-affine aggregate with a growing exponent $\beta_{\rm EM} = 1/3$ and a roughness exponent $\alpha_{\rm EM} = 1/2$. A generalization of the EM is proposed and studied, such that the growing probability now depends on the distance to the interface. This new model is called the unstable Eden model (UEM) because, within the short time regime, it exhibits an unstable growth mode with a growing exponent $\beta_{\rm UEM} > 1/2$. However, in the asymptotic time regime the interface becomes stabilized and a roughness exponent $\alpha_{\rm UEM} = 1$ can be defined. In contrast to the EM, the interface of the UEM is no longer self-affine. Based on extensive numerical simulations it is concluded that the interface generated by the collision between the EM and the UEM is characterized by a roughness exponent $\alpha_{\rm coll} = \alpha_{\rm UEM} = 1$.

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1. Introduction

The study and understanding of the properties of growing interfaces has recently developed into a very active field of multidisciplinary research [1–6]. This interest is due to the fact that evolving interfaces are present in a wide variety of physical, chemical and biological systems and processes such as film growth by either molecular beam epitaxy, vapour deposition or chemical deposition [1,2], propagation of fire fronts [7], bacterial growth [8], solidification [9], propagation of reaction fronts in catalyzed reactions [10, 11], electrodeposition/dissolution experiments [12], ballistic deposition [13], sedimentation [14], etc.

While different growth morphologies, such as layer by layer growth [15], unstable growth [6], etc, have been identified, the development of self-affine surfaces is, so far, the most frequently encountered growth process [1–6]. In a self-affine growing aggregate the orientation of the interface is maintained essentially parallel to the substrate, but it becomes rough. The amount of roughness increases with time and, on a large observation scale, it evolves in a self-affine fashion [5]. In contrast, unstable growth is observed when the initially selected orientation of the substrate can no longer be maintained [5].

The aim of this paper is to study the properties of the interface generated by the collision of two interfaces of different roughnesses. We define that a collision event takes place when the last occupied growing site of one interface has at least one occupied neighbour belonging to the other interface. So, the set of growing sites having that characteristic defines the collision interface [16].

Our study is motivated by various physical processes such as the collision of fire fronts in burning experiments or upon forest-fire evolution, solidification of immiscible fluids, interference of reaction fronts during the propagation of chemical waves, etc. In a previous paper, Albano [16] investigated the interface generated by the collision of two Eden clusters. In that case, the resulting interface has the same roughness exponent as the colliding interfaces before the collision. That paper posed an interesting question on the roughness of the interface generated by two colliding interfaces of different roughnesses. In fact, *a priori*, it is not clear whether the characteristics of the new interface would be dominated by either the rougher or the smoother surface. Eventually, an interface characterized by a new roughness exponent could emerge.

It should be noted that our study on the collision of two interfaces is essentially different from the work of Derrida and Dickman [17] since that work focused on the investigation of the interface generated by a competitive growth process.

In addition to the primary scope of this paper, a variation of the Eden model (EM), called the unstable Eden model (UEM), is proposed and studied. It turns out that the interface of the UEM is no longer self-affine, as in the case of the EM. Furthermore, both models have different roughness exponents and consequently they are appropriated for the investigation of the properties of a collision interface generated by two growing systems of different characteristics.

The paper is organized as follows: in section 2 a brief theoretical background of dynamic scaling theory applied to self-affine interfaces is outlined. In section 3 the EM is described while the UEM is presented. Section 4 is devoted to the study of UEM while in section 5 results corresponding to the collision between the interfaces of both EM and UEM are presented and discussed. Finally, our conclusions are stated in section 6.

2. Brief background on the dynamic scaling approach

The phenomenological scaling approach to the dynamic evolution of a self-affine interface was developed earlier by Family and Vicsek [4, 18] and has become a useful tool to characterize self-affine roughness. Considering a flat, $(d-1)$-dimensional, surface at time $t = 0$ and paying attention to the growing process that occurs essentially parallel to the surface, it is possible to assume, without loss of generality, that there exists a well defined growth direction and that the surface can be described by a function $h(x, t)$ which gives the height of the interface at time t and position x. Of course, such a height is measured from the initial flat surface at $t = 0$. If the interface could not be described by a single valued function of x, the function $h(x, t)$ gives the maximum height of the interface at x. Considering a section of the surface having a typical size L (in each of the $(d - 1)$ dimensions of the surface) the average height of the surface at time t is defined as

$$\langle h(t)\rangle = \frac{1}{(L^{d-1})}\sum_{x} h(x, t) \qquad (1)$$

where the summation runs over all x. The interface width $w(L, t)$ at time t may be defined by the rms of the height fluctuations given by

$$w(L,t) = \left(\langle h^2(t)\rangle - \langle h(t)\rangle^2\right)^{1/2}. \tag{2}$$

The Family–Viscek scaling approach assumes that

$$w(L,t) = L^{\alpha} F(t/L^{z}) \tag{3}$$

where $F(x) \propto x^{\beta}$ for $x \ll 1$ and $F(x) \to$ constant for $x \gg 1$, with $z = \alpha/\beta$. Also α, β and z are the roughness, growing and dynamic exponents, respectively.

The dynamic exponent z describes the evolution of the correlated region with time: initially different parts of the surface are independent, but regions of correlated roughness form over time and their size increases as $\xi \propto t^{1/z}$. In each correlated region the width of the surface increases as the observation scale raised to the growing exponent α. Thus, for a finite sample of side L and $t \to \infty$ the width of the growing interface reaches a statistically stationary state such as $w(L) \propto L^{\alpha}$. Furthermore, the overall width of the interface increases as t^{β} until it reaches a maximum of the order L^{α}.

3. The Eden model and the unstable Eden model

The EM was introduced earlier by Eden [19] as a growth model for tumour cells. Since then the EM has become a prototypical growth model. The EM is defined here in a square lattice of width L and length M in (1+1) dimensions. The sites of the lattice are labelled by indices (i, j) such as $1 \leqslant i \leqslant L$ and $1 \leqslant j \leqslant M$. The growing process starts from a row of occupied sites at $j = 1$ $(\forall i)$ while the remaining sites of the lattice are left vacant. The Eden clusters grow simply by adding new particles to perimeter sites, i.e. those sites that are nearest neighbours of already occupied sites. Specifically, the EM assumes that all perimeter (or growing) sites have the same probability of becoming occupied.

Eden clusters are compact objects with a self-affine interface characterized, in (1 + 1) dimensions, by exponents $\beta_{\mathrm{EM}} = 1/3$ and $\alpha_{\mathrm{EM}} = 1/2$, and consequently $z_{\mathrm{EM}} = \alpha_{\mathrm{EM}}/\beta_{\mathrm{EM}} = 3/2$ [1].

To investigate the properties of the interface generated by the collision between two interfaces of different roughnesses the UEM is proposed. In the UEM the growing sites have a probability $P(j - \langle h\rangle)$ of becoming occupied, such as

$$P(j - \langle h\rangle) = C\,|j - \langle h\rangle|^{\delta} \tag{4}$$

where δ is an exponent that can be tuned as an external parameter and $\langle h\rangle$ is the position of the growing interfaces as given by equation (1). It should be noticed that, after each deposition event, the growing probability of all perimeter sites has to be evaluated, such that the normalization constant becomes $C = 1\big/\sum_{\mathrm{allsites}} P(j - \langle h\rangle)$. Also, notice that for $\delta = 0$ the UEM gives the classical EM. The UEM is inspired in the experimental observation of the growth of unstable interfaces upon both chemical vapour deposition of SiO_2 [20] and electrodeposition of Cu [21]. In fact, these experiments show that, due to the preferential deposition of atoms in some regions of the sample, one observes the development of protrusions surrounded by deep valleys and the growth of the interface becomes unstable during a transient period. This kind of structure can be observed in the snapshot configuration of the UEM shown in figure 1. However, in the experiments and for the long time regime, the surface reaches a scale-invariant stationary state compatible with the Kardar–Parisi–Zhang universality class [20]. The UEM explicitly takes into account this preferential deposition through the deposition probability given by equation (4) and successfully describes the experimental findings [22].

Figure 1. Typical snapshot configuration of the growing interface of the UEM within the width saturation regime, obtained using a sample of width $L = 256$.

4. Numerical study of the UEM

The UEM is studied in the square lattice of width L, with $24 \leqslant L \leqslant 1536$, by means of Monte Carlo simulations. The Monte Carlo time step (MCS) is the time unit and corresponds to the deposition of L particles. Results are averaged over 1500–5000 different runs, depending on the size of the lattice. The probability $P(j - \langle h \rangle)$ given by equation (4) has to be evaluated after every deposition event, since such an event may cause both a displacement of the interface location $\langle h \rangle$ and a change in the number of growing sites.

Figure 2(a) shows log–log plots of $w(t, L)$ versus t obtained for the UEM with $\delta = 1$ and using lattices of different sizes. The broken line has slope $\beta_{\mathrm{UEM}} = 2/3$ and has been drawn for the sake of comparison. In fact, a least-squares fit of the short time behaviour of $w(t)$ (for $0.001 \leqslant t \leqslant 1$) gives[1] $\beta^*_{\mathrm{UEM}} = 0.70 \pm 0.05$. In contrast, the EM has a growth exponent $\beta_{\mathrm{EM}} = 1/3$. It should be noticed that growth models with $\beta > 1/2$ are generally called unstable because growth occurs when the initially selected orientation of the substrate can no longer be maintained [5]. This kind of phenomena can clearly be observed in the snapshot configuration shown in figure 1, where the lateral interfaces of the protrusion run almost perpendicularly to the horizontal substrate. It should be stressed that this short time (transient) instability becomes healed in the asymptotic regime, such as the interface width becoming saturated, as shown in figure 2(a), reaching an L-dependent value ($w_s(L)$). The stabilization of the interface is due to the operation of the following feedback mechanism: the growth starts from a flat surface and subsequently the unstable growth regime is triggered so that the interface is characterized by a hill surrounded by valleys (see, e.g., figure 1). Recalling that the growing probability depends on the distance to the average position of the interface (equation (4)), one has that, after the transient period, such a probability is almost the same for both the top of the hill and the bottom of the valleys. If, for example, a stochastic fluctuation causes a hill to grow up higher than the average, then the average position of the interface

[1] Hereafter we will use the superscript ($*$) to identify the values of the exponents obtained by least-squares fits. Such a symbol will be omitted to denote the conjectured values of the exponents.

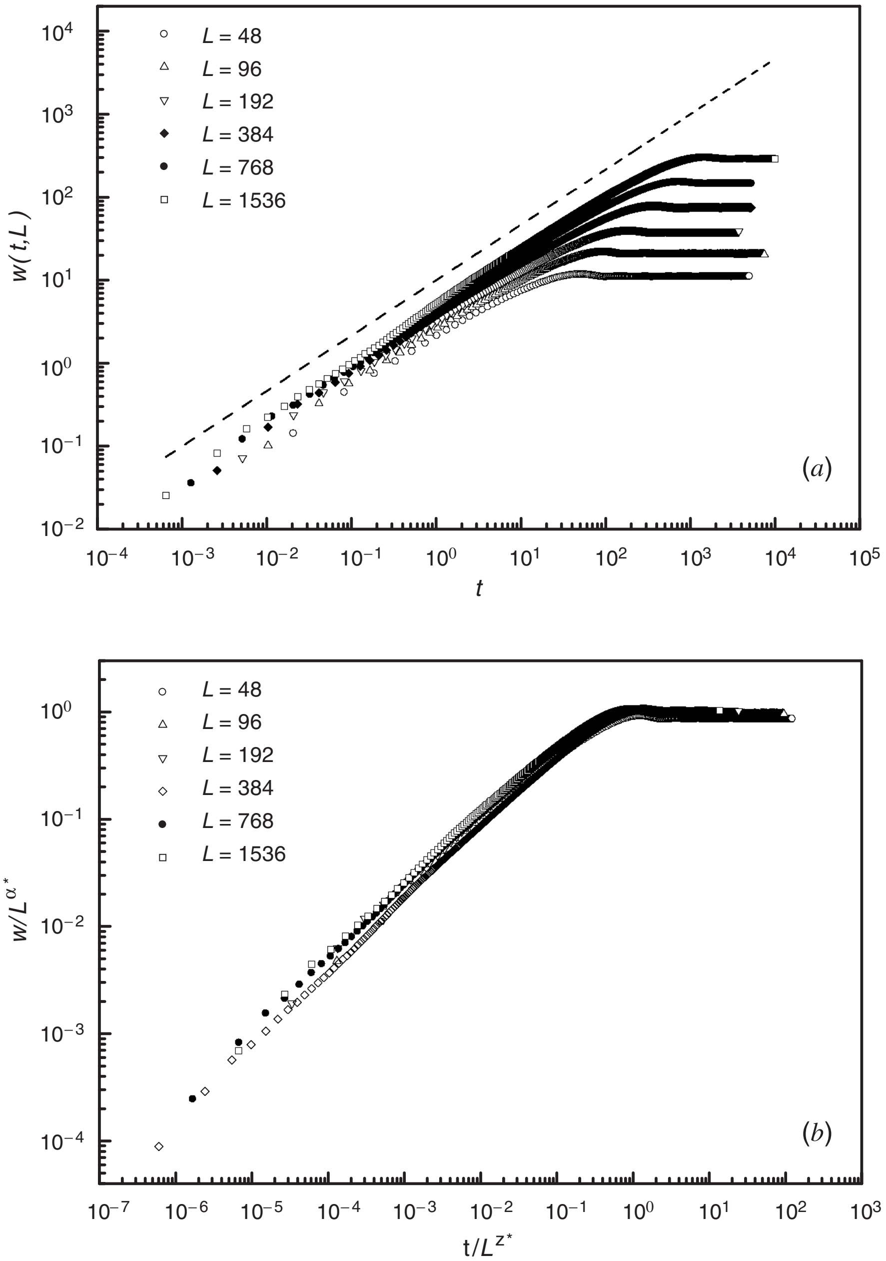


Figure 2. (a) Log–log plots of $w(t, L)$ versus t obtained for the UEM with $\delta = 1$ and using lattices of different sizes L, as indicated in the figure. The broken line has slope $\beta_{\mathrm{UEM}} = 2/3$ and has been drawn for comparison. (b) Scaling plot of $w(t, L)/L^{\alpha^*}$ versus t/L^{z^*}, according to equation (3).

will shift upwards, causing an enhancement of the growing probability of sites located at the bottom of the valleys. The subsequent deposition on the valleys will cause the suppression of the fluctuation and the stabilization of the interface. Of course, the same mechanisms operates if the fluctuation starts in the valleys.

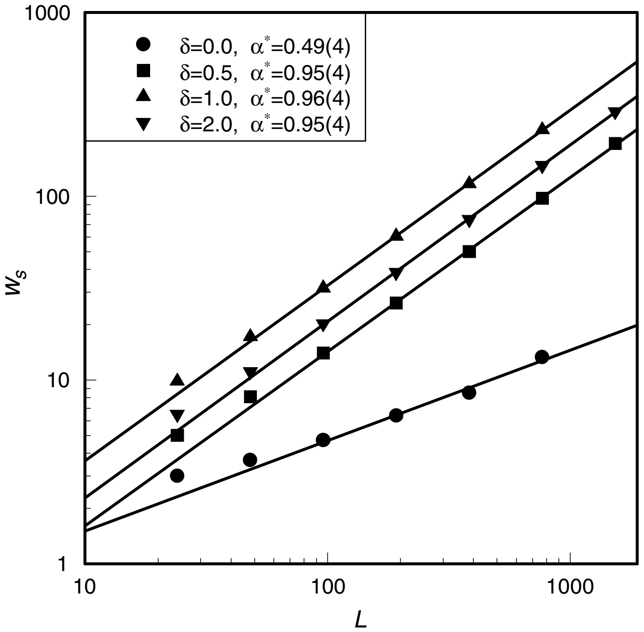


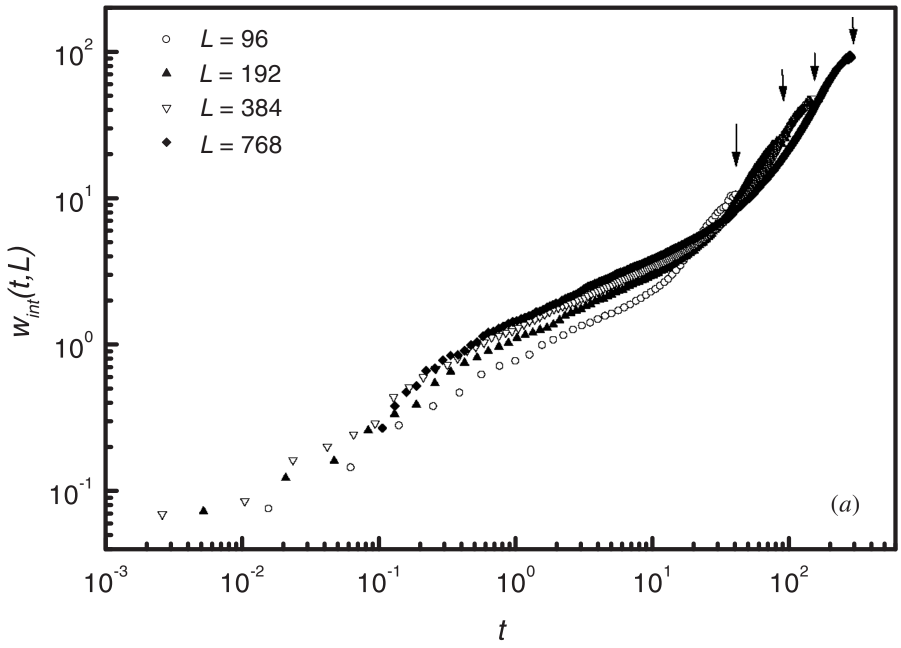
Figure 3. Log–log plots of $w_s(L)$ versus L for both EM ($\delta = 0$) and UEM with different values of δ as indicated in the figure.

The stabilization of the interface, as already discussed, allows us to define the roughness exponent α. In fact, figure 3 shows log–log plots of $w_s(L)$ versus L obtained for both EM and UEM. For the case of the EM ($\delta = 0$) the well known roughness exponent $\alpha_{\mathrm{EM}} = 1/2$ is obtained (a least-squares fit gives $\alpha^*_{\mathrm{EM}} = 0.49 \pm 0.04$). However, for the case of the UEM and using different values of the parameter δ (see equation (4)), one obtains values of the roughness exponent that are independent of δ, with $\alpha_{\mathrm{UEM}} \cong 1$ ($\alpha^*_{\mathrm{UEM}} = 0.96 \pm 0.04$). Also, using $z = \alpha/\beta$, one has $z^*_{\mathrm{UEM}} = 1.37 \pm 0.15$ for the dynamic exponent.

Figure 2(*b*) shows log–log plots of the scaled width versus the scaled time, as suggested by equation (3). The excellent data collapse that is observed strongly suggests that the UEM satisfies the Family–Vicsek scaling ansatz [4, 18] with the obtained exponents $z^*_{\mathrm{UEM}} = 1.37$ and $\alpha^*_{\mathrm{UEM}} = 0.96$. It should be noticed that at least two different mechanisms can lead to such dynamic scaling for the overall interface width. One is a self-similar growth, while the second is a coarsening process in which a single feature (e.g. a protrusion) becomes dominant. In the first case, there is self-similarity as reflected in spatial homogeneity. However, in the second case there is no spatial homogeneity and the profile varies from place to place. The latter behaviour holds for the UEM, as follows from the observation of the snapshot of figure 1, and consequently the UEM interface is no longer self-affine.

Models with $\alpha > 1$ exhibit super-roughening since the density of sites of the interface diverges in the thermodynamic limit [23, 24]. So, the proposed UEM with $\alpha_{\mathrm{UEM}} \cong 1$ may be marginally super-rough.

In view of these results, it is also concluded that, for $\delta > 0$, the exponents of the UEM are independent of δ, at least within the range of the parameter that has been studied. Furthermore, in contrast to the classical EM, the UEM interface lacks self-affinity.



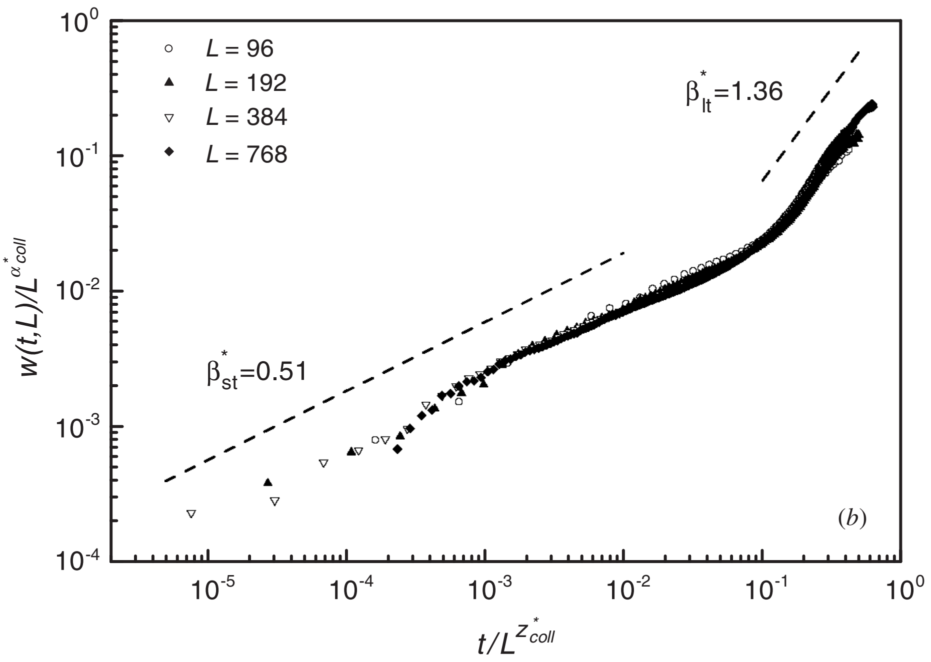


Figure 4. (*a*) Time evolution of the colliding interface width $w_{\rm int}(t, L)$ obtained for lattices of different sizes L as indicated in the figure. The arrows show the location of the maximum collision time $t_{\rm max}$. (*b*) Master curve $w_{\rm int}(t, L)/L^{\alpha^*_{\rm coll}}$ versus $t/L^{z^*_{\rm coll}}$. The broken lines have been drawn for comparison.

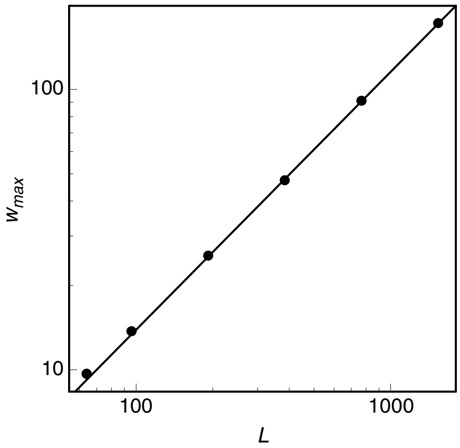


Figure 5. Log–log plot of $w_{\rm max}$ versus L for the colliding interface. The straight line has slope $\alpha^*_{\rm coll} = 0.96$ and corresponds to the least-squares fit of the data.

5. Properties of the interface generated by the collision between the EM and the UEM

Let us recall that it is considered that a collision event takes place when the last occupied growing site of one cluster has at least one occupied neighbour belonging to the other cluster. In this way, the collision interface is defined as the set of that kind of growing sites [16]. In order to follow the collision process we first allow both the EM and UEM colliding interfaces to reach saturation, i.e. for $t \gg L^z$. Subsequently, when the first collision event is detected the time scale is initialized to $t = 0$ and the time evolution of the collision interface is measured. The collision stops when all growing sites of both clusters become exhausted.

Figure 4(a) shows the width of the colliding interface ($w_{\rm int}$) as a function of time, obtained for lattices of different sizes. Two different time regimes can be observed, as will be discussed in detail below.

From the data shown in figure 4(a) one can evaluate the average maximum width of the interface due to the collision ($w_{\rm max}$) and a log–log plot of $w_{\rm max}$ versus L, as is shown in figure 5, which gives a straight line with slope $\alpha^*_{\rm coll} \cong 0.96 \pm 0.05$ that is consistent with $\alpha_{\rm coll} = 1$. Therefore, the interface resulting from the collision of two interfaces of different roughnesses can be characterized by the roughness exponent of the rougher interface. It should be noted that previous simulations have shown that when the colliding interfaces have the same roughness the resulting interface also adopts the roughness exponent of such interfaces [16].

Also, using the data of figure 4(a) it is possible to evaluate the maximum (averaged) time ($t_{\rm max}$) elapsed during the collision process (see, e.g., the arrows in figure 4(a)). A log–log plot of $t_{\rm max}$ versus L (not shown here for the sake of space) indicates that the relationship $t_{\rm max} \propto L^{z_{\rm coll}}$ holds with a dynamic exponent for the collision given by $z^*_{\rm coll} = 0.95 \pm 0.05$ that is consistent with $z_{\rm coll} = 1$.

After obtaining the exponents $\alpha_{\rm coll}$ and $z_{\rm coll}$ it is possible to drawn a re-scaled plot of $w_{\rm int}$ versus t as is shown in figure 4(b). The satisfactory data collapse obtained allows a clear distinction to be made between the two different time regimes, as was earlier anticipated by figure 4(a). In fact, there is a short time behaviour with $\beta^*_{st} = 0.51 \pm 0.01$ and a long time behaviour with $\beta^*_{lt} = 1.36 \pm 0.05$. The crossover time is close to $t_{\rm cross}/L \approx 0.1$. Since for the short time regime the growing exponent is very close to that corresponding to a random deposition process, $\beta_{RD} = 1/2$, our finding may be understood if the collision interface starts to grow with sporadic (randomly distributed) collision contacts between the Eden interface and the topmost sites of the UEM interface. Subsequently, for $t/L \gtrsim t_{\rm cross}$, the growth process becomes dominated by collisions between the standard Eden interface and the deep valleys characteristic of the UEM. Consequently, the resulting interface is highly unstable with $\beta_{lt} \gg \beta_{RD}$. Notice that the observed data collapse indicates that, not only $t_{\max} \propto L^{z_{\mathrm{coll}}}$ but also the crossover time, scales with the same behaviour, namely $t_{\mathrm{cross}} \propto L^{z_{\mathrm{coll}}}$.

6. Conclusions

A new EM, such as the growing probability of a perimeter site is proportional to the distance to the actual position of the interface, is presented and studied. The model exhibits a short time instability: however, in the long time regime the interface becomes stable. Such an interface is no longer self-affine. The fact that $\alpha_{\mathrm{UEM}} = 1 > \alpha_{\mathrm{EM}} = 1/2$ provides the possibility of studying the properties of the interface resulting from the collision of interfaces of different roughnesses. Our numerical results give strong evidence that the colliding interface adopts the roughness exponent of the rougher interface. We expect that this numerical prediction may be tested both experimentally and theoretically since the properties of interfaces are of great interest in many physical, chemical and biological systems.

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